Signaling in Sovereign Debt Markets

VERY PRELIMINARY AND INCOMPLETE: DO NOT CIRCULATE

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Abstract

We present a model of asymmetric information in which lenders infer persistent, hidden sovereign types from both borrowing and default behavior. Sovereigns come in two types with different proclivities to default. The Markov Perfect Bayesian Equilibrium features hybrid pooling, in which different types pool along the borrowing margin but separate along the default margin. This is a result of strong general equilibrium effects that attenuate differences across types: The bad type receives a better price schedule, which drives him to more frequent repayment, while the good type receives a worse price schedule, which drives him to more frequent default. This makes the price schedule relatively flat in beliefs. Consequently, the good type has little incentive to distinguish himself via higher borrowing and thus does not do it.

1 Introduction

Information frictions have long been known to plague markets for international debt e.g. Cole and Kehoe (1998), Chari and Kehoe (Chari and Kehoe), Angeletos and Werning (2006), Cole et al. (2016), Carlson and Hale (2006), Van Nieuwerburgh and Veldkamp (2009), or Gu and Stangebye (2018) to name only a few. Investors typically do not have access to all of the information that they would like to have at the time they make the investment decision, largely because the borrowing country is foreign in nature.

Despite the vast literature exploring different facets of informational asymmetries, to the authors' knowledge a general recursive signaling game between a sovereign government and its lenders in which all actions can be interpreted as signals has not been studied. Default alone as a signal has been extensively studied (see Cole and Kehoe [1998], Sandleris [2008], Phan [2017a, 2017b], or Amador and Phelan [2018]). In this paper we undertake this endeavor and study a quantitative model in the vein of Arellano (2008) or Aguiar

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et al. (2016) in which foreign lenders do not observe domestic shocks, but the sovereign does. A proper signaling game is exposited in which both default and fiscal behavior, e.g., borrowing serve as signals. Interestingly, we find as a result what these papers typically assume: Namely that default emerges as an information-conveying signal even when other options are on the table.

In the model, there is a good type of sovereign and a bad type. The bad type is more myopic and thus has a tendency to default more often. Types are persistent, but unobserved by lenders. There are also unobserved transitory shocks that further obfuscate inference. Lenders formulate beliefs, placing some weight on the possibility that the sovereign is the bad type and pricing debt issuance accordingly. There are also output shocks observed by all agents to make the model more realistic and related to the quantitative literature.

In the only equilibrium that we can compute, we find a couple of novel and interesting implications. First, the equilibrium features hybrid pooling. That is, the good and bad types pool borrowing behavior but separate at default. This is due to a strong general equilibrium effect that arises in the model: The price can only depend on beliefs, not the actual type. Thus, the good type and the bad type share a common price schedule conditional on beliefs. This shared price schedule is typically worse than the good type's and better than the bad type's. Thus, debt is more expensive to service for the good type and cheaper to service for the bad type, which causes the good type to default more and the bad type to default less. This attenuation in default behavior brings together the endogenous 'credit limit'[1] beyond which lenders refuse to lend, until it is the same for both types.

Despite sharing a common credit limit, the bad type is still more myopic and ceteris paribus, tends to default more often. Thus, there is still separation at default and this translates to prices: The more weight lenders place on the sovereign being the good type, the higher the price will be. This discrepancy is not as large, however, as it is across the types when there is no switching or asymmetry of information. In particular, it is not large enough to incentivize the good type to distinguish himself via extra borrowing and so he never does so. Hence the pooling remains.

The model has interesting implications for the trajectory of beliefs over time. It suggests that defaulting immediately downgrades lender beliefs i.e. increases their prior that the sovereign is the bad type. Following

[1]By 'credit limit,' we mean the relative 'cliff' in the downsloping debt demand schedule as a function of issuance.

re-entry, however, beliefs become more optimistic over time: As both types pool behavior, the longer the sovereign goes without default, the greater the chance that he is the good type. This accords with the finding of many other papers, e.g., Cole and Kehoe (1998) or Amador and Phelan (2018), who have more restricted signal spaces.

2 Model

2.1 Shocks and Information Structure

We begin with a model with two types of shocks: Those that are observable to all parties and those that are only observed by the sovereign. We assume that all parties are privy to realizations of the country's growth rate, and that this follows a stochastic process following Aguiar and Gopinath (2006). In particular, $Y_t = Y_{t-1}e^{g_t}$ and

$$g_t = (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \sigma_g \epsilon_t$$

where $\bar{g}$ is the average growth rate, $\rho_g \in (0,1)$ is its persistence, σ_g is its conditional volatility, and ϵ_t is an iid standard normal. We will bundle these observable states into a vector $s_t = (Y_t, g_t)$.

In addition to these, there are two shocks unobservable to the lenders: A binary, persistent sovereign type, which follows a Markov process, and a transitory shock. We will assume that the former is the sovereign discount factor, i.e., $\beta \in \{\beta_L, \beta_H\}$; Aguiar et al. (2016) argue that this parameter is pivotal in governing default propensities.

The transitory shock is iid and can be interpreted either as an endowment shock or a preference shock to the level of subsistence consumption, which directly affects the sovereign's incentives to borrow.[2]

We assume that $0 < \beta_L < \beta_H < 1$ and that there is a symmetric transition probability and that the process is persistent, i.e.,

$$Pr(\beta_H|\beta_L) = Pr(\beta_L|\beta_H) = p < 0.5$$

We will refer to β_L as the 'bad' type, since this type will generally suffers from more severe commitment

[2] See Chatterjee and Eyigungor (2012) for a proof of this result.

problems than β_H. The transitory shock has any potentially type-specific distribution, $m_t = Y_t m$, where $m \sim F_\beta$. We assume that it is continuous, iid, and cointegrated with the sovereign endowment.[3]

This dual shock structure serves several purposes. First and foremost, it is empirically plausible. When lenders see a sudden surge in indebtedness or default risk, they are often left trying to tease apart how much is due to underlying institutional change in the borrowing country and how much is due to random, uncontrollable liquidity shocks that can happen even to a well-behaved government. Policy debate in the early days of nearly any debt crisis, from the Latin American debt crisis in the 1980's to the recent Eurozone crisis, often centers around these two competing frameworks.

Second, this dual-shock structure give the inference problem a smoothness that is conducive to computation: Minor changes in one type's behavior will typically not lead to discrete fluctuations in beliefs. This makes the computation tractable and allows for quantitative analysis.

Third, it provides hidden information the opportunity to influence both default and borrowing incentives. Even allowing for this, we will see that, in equilibrium, only default emerges as the signal that conveys information. We do not find, for instance, that default signals the discount factor whereas borrowing reveals the transitory shock.

The lenders will attempt to infer the sovereign's type from his borrowing and default behavior. We will assume that their beliefs at any time t are summarized by the scalar

$$\rho_t = Pr(\beta = \beta_L | x^t)$$

Every period, the lenders update their beliefs using Bayes' rule according to x^t, which is the lenders' entire information set at time t, which we will assume to be the history of all borrowing and default decisions as well as prices.

We will bundle these unobserved states together into a vector $u_t = (\beta_t, m_t)$. The debt stock is observed by all parties, but for expositional clarity we do not bundle it with the other observed states.

[3]Since m_t is iid, lenders will not need to carry around beliefs regarding its value. Given ρ_t, beliefs regarding it could be inferred, but they will never be used in forecasting or demand. For expositional simplicity, we omit it entirely.

2.2 Sovereign

The sovereign borrower suffers from limited commitment. In period t, he cannot commit to either borrowing or default behavior in period $t+1$. He has a time-separable utility function, $u(\cdot)$, and is a monopolist in his own debt market.

The sovereign will behave much as he would in a standard sovereign default model model e.g. Aguiar and Gopinath (2006) or Arellano (2008). We will assume as in Chatterjee and Eyigungor (2012) that the sovereign chooses debt issuance from a discrete set. This implies that his problem can be expressed recursively in a Bellman equation conditional on repayment of the current debt stock, B_t:

$$
\begin{aligned}
V_{R,t}(s_t, u_t, \rho_t, B_t) = \max_{B_{t+1}\in\mathcal{B}} & \, u\left(C_t - m_t\right) + \\
& \beta_t E_t \left[\max\{V_{R,t+1}(\tilde{s}_{t+1}, \tilde{u}_{t+1}, \rho_{t+1}, B_{t+1}), V_{D,t+1}(\tilde{s}_{t+1}, \tilde{u}_{t+1}, G_{D,t+1}(\tilde{s}_{t+1}, \rho_{t+1}, B_{t+1}))\}\right] \\
\text{s.t. } \; C_t = & \; Y_t - B_t + q_t(B_{t+1}|s_t, \rho_t, B_t)B_{t+1} \qquad (1) \\
\rho_{t+1} = & \; H_t(s_t, \rho_t, B_t, B_{t+1})
\end{aligned}
$$

H_t is a belief updating rule that the sovereign takes as given that will be described momentarily. $G_{D,t}$ is a belief updating rule that is employed following a sovereign default, which reveals other information. Both are taken as given by the sovereign.[4] Generally ρ_{t+1} need not equal $\rho_{D,t+1}$. For now, all that matters is that the sovereign *could* influence it through his debt issuance or default decision. We will discuss its formal shape momentarily.

Interestingly, the value of default depends on the current level of indebtedness even though there is zero recovery. This is because the size of a default it may reveal additional information about the sovereign's type, and this information/reputation follows him into the period of exclusion.

If the sovereign defaults he is excluded from credit markets temporarily, receiving a value $V_{D,t+1}$, which entails no m-shocks (they are set to the average) but a constant output cost, ϕ. There will be exogenous re-entry with zero debt obligations. Thus, the Bellman for default is

[4] Since $G_{D,t}(\rho_t, B_t)$ is taken as given, we can express the default policy function, which will depend on $G_{D,t}$, as a function (s_t, u_t, ρ_t, B_t).

$$V_{D,t}(s_t, u_t, \rho_t) = u(Y - \psi - \bar{m}) + \beta_t E_t \left[(1-\phi) V_{D,t+1}(\tilde{s}_{t+1}, \tilde{u}_{t+1}, \rho_{t+1}) + \phi \max\{V_{D,t+1}(\tilde{s}_{t+1}, \tilde{u}_{t+1}, \rho_{t+1}), V_{R,t+1}(\tilde{s}_{t+1}, \tilde{u}_{t+1}, \rho_{t+1}, 0)\}\right] \tag{2}$$

$$\rho_{t+1} = p + (1-2p)\rho_t$$

Notice that, once in default, the type and belief process evolve exogenously with no chance for information to influence beliefs. This is because the sovereign takes no actions in default.

We also assume that the sovereign defaults if it is not possible for him to raise enough revenue to satisfy liquidity needs i.e. if $\max_{B_{t+1}} q_t(B_{t+1}|s_t, \rho_t, B_t)B_{t+1} < m_t + B_t - Y_t$.

2.3 Belief Updating

In equilibrium, lenders update their beliefs regarding β every period in response to sovereign borrowing and default behavior. They do so optimally, i.e., according to Bayes' rule. To define this, let $A_t(s_t, u_t, \rho_t, B_t)$ be the sovereign's borrowing policy function in equilibrium. In states of repayment, beliefs will be updated as follows:

$$\begin{aligned} H_t(s_t, \rho_t, B_t, B_{t+1}) &= G_t(s_t, \rho_t, B_t, B_{t+1})(1-p) + (1 - G_t(s_t, \rho_t, B_t, B_{t+1}))p \\ &= p + (1-2p) G_t(s_t, \rho_t, B_t, B_{t+1}) \end{aligned} \tag{3}$$

where $G_t(s_t, \rho_t, B_t, B_{t+1})$ is the optimal inference of the *current* probability $\beta = \beta_L$ based on the sovereign's actions in period t. For a given (s_t, ρ_t, B_t), define $M^i(s_t, \rho_t, B_t, B_{t+1}) = \{m | A_t(s_t, (\beta_i, m), \rho_t, B_t) = B_{t+1}\}$. The belief-updating function is

$$G_t(s_t, \rho_t, B_t, B_{t+1}) = \begin{cases} 1, & M^i(s_t, \rho_t, B_t, B_{t+1}) \text{ empty for } i \in \{L, H\} \\ \frac{\rho_t \int_{m \in M^L(s_t, \rho_t, B_t, B_{t+1})} f_{\beta_L}(m) dm}{\rho_t \int_{m \in M^L(s_t, \rho_t, B_t, B_{t+1})} f_{\beta_L}(m) dm + (1-\rho_t) \int_{m \in M^H(s_t, \rho_t, B_t, B_{t+1})} f_{\beta_H}(m) dm}, & o/w \end{cases}$$

Thus, if it's the case that *only* the β_L-type would ever issue B_{t+1}, the lenders instantly believe that he's of that type; the same is true if there is some issuance that could only the β_H-type. However, if it's possible that this issuance could have come from either of the two types, the lenders update their beliefs using

Bayes' rule based on the relative likelihood of the given issuance in each case.

When the sovereign defaults, lenders perform a similar inference. Denote the default policy by $D_t(s_t, u_t, \rho_t, B_t)$. We again define a subset of the domain of the m-shock by $M_D^i(s_t, \rho_t, B_t) = \{m | D_t(s_t, (\beta_i, m), \rho_t, B_t) = 1\}$. $G_{D,t}(s_t, \rho_t, B_t)$ describes the immediate window in which investors update their beliefs following a default: The first argument is the current beliefs were the sovereign to repay and the second is the current level of debt. Bayes' rule implies

$$G_{D,t}(s_t, \rho_t, B_t) = \begin{cases} 1, & M_D^i(s_t, \rho_t, B_t) \text{ empty for } i \in \{L, H\} \\ \frac{\rho_t \int_{m \in M_D^L(s_t,\rho_t,B_t)} f_{\beta_L}(m)dm}{\rho_t \int_{m \in M_D^L(s_t,\rho_t,B_t)} f_{\beta_L}(m)dm + (1-\rho_t) \int_{m \in M_D^H(s_t,\rho_t,B_t)} f_{\beta_H}(m)dm}, & o/w \end{cases}$$

We assume for now that off-equilibrium actions (borrowing and default) are met with the belief that the sovereign is the bad type. This helps with computation and is a legitimate equilibrium concept, though it might not satisfy the intuitive criterion.

2.4 Lenders

We assume that lenders are competitive, risk-neutral, and deep-pocketed. They have access to a risk-free return of r against which they price default risk. As was described before, they have beliefs over the value of β_t that are summarized in ρ_t. Given their beliefs, they price default risk. This implies the following pricing function:

$$q_t(B_{t+1} | s_t, \rho_t, B_t) = \frac{1}{1+r} E_{\tilde{s}_{t+1}, \tilde{u}_{t+1} | \rho_t} \left[\mathbf{1}\{V_{R,t+1}(\tilde{s}_{t+1}, \tilde{u}_{t+1}, \rho_{t+1}, B_{t+1}) \geq V_{D,t+1}(\tilde{s}_{t+1}, \tilde{u}_{t+1}, G_{D,t+1}(\tilde{s}_{t+1}, \rho_{t+1}, B_{t+1}))\} \right] \quad (4)$$

Notice that this expectation will depend on the equilibrium belief updating rule, H_t.

2.5 Equilibrium Definition

Definition 1. *A* ***Markov Perfect Bayesian Equilibrium (MPBE)*** *is a set of value, policy, belief updating, and price functions such that Recursions 1 and 2 and Equations 3 and 4 are satisfied.*

We will say that an MPBE is *pooling along the borrowing margin* if $A_t(s_t, u_t^1, \rho_t, B_t) = A_t(s_t, u_t^2, \rho_t, B_t)$ for any feasible pair (u_t^1, u_t^2) and for all t. We will say that an MPBE that is not pooling is *separating.*

Symmetrically, we will say it is *pooling along the default margin* if $D_t(s_t, u_t^1, \rho_t, B_t) = D_t(s_t, u_t^2, \rho_t, B_t)$ for any feasible pair (u_t^1, u_t^2) and for all t.

There could be many Markov Perfect Bayesian Equilibria with varying properties. We will focus on those for which we can robustly solve numerically. All of these share the same basic properties that we will exposit briefly.

2.6 Quantitative Implementation

We solve the model using iterative methods. We follow Chatterjee and Eyigungor (2012) and treat m_t as a continuous shock and compute thresholds over it for varying discrete debt levels. These thresholds also prove useful in the belief-updating rules.

In the benchmark model, we set $r = .01$, $p = .01$, $\beta_L = 0.8$ and $\beta_H = 0.85$. We also assume that the distribution of m_t does not depend on β_t, but rather follows a mean-zero normal distribution truncated at three standard deviations with a standard deviation $\sigma_m = .003$, which is its value in Chatterjee and Eyigungor (2012). We also assume that the sovereign has power utility with $\sigma = 2.0$. We also set $\phi = 0.0625$ i.e. we assume average re-entry is ever 4 years.

Our growth process is taken from Aguiar and Gopinath (2006): $\mu_g = .01$, $\rho_g = .17$, and $\sigma_g = .03$. We tauchnize this process over 25 discrete points.

Our grid over debt is equally spaced and ranges from $[0, .2]$ with $|\mathcal{B}| = 21$. We use linear interpolation over 11 discretized points for lender beliefs, ρ_t. For m_t, we use the CDF to compute default probabilities and belief-updating functions and take expectations using the trapezoidal method when necessary employing 11 discretized points.

The results are robust to a wide choice of parameters with the exception of the grid size over debt. In order for computation to be feasible, we typically require that $|\mathcal{B}|$ be fairly low. Our conjecture is that equilibria may not exist for finer debt grids: Small changes is debt levels may update beliefs, which may change incentives to issue in a non-convergent way.

We do not necessarily see this as unrealistic. In reality, bond issuances are often discrete in nature and investors typically only care about differences across fairly large quantities e.g. 400 million euros versus 500 million euros. There are models of bounded rationality in which such behavior is plausible (Matejka and Sims [2009], Armenter et al. [2018]), but they will not be modeled here.

3 Results

It will be easiest to understand the results if we first look at the two cases in which no regime-switching takes place. Debt demand schedules in steady state are provided in Figure 1. Notice that in both cases there is a endogenous 'cliff' in the pricing function beyond which the sovereign cannot be trusted to repay. The sovereign is quite impatient in both cases and so the motive to front-load consumption overrides the motive to smooth consumption against shocks to m_t. This implies that the policy function is essentially determined by the sovereign borrowing up to the cliff. Since the bad type is more myopic, he defaults more often and gets a worse pricing schedule. For this reason, the bad type borrows less in the absence of switching: $A_L(\bar{g}, m, B) = 0.03$ and $A_H(\beta, m, B) = 0.05$ for any (m, B) pair.

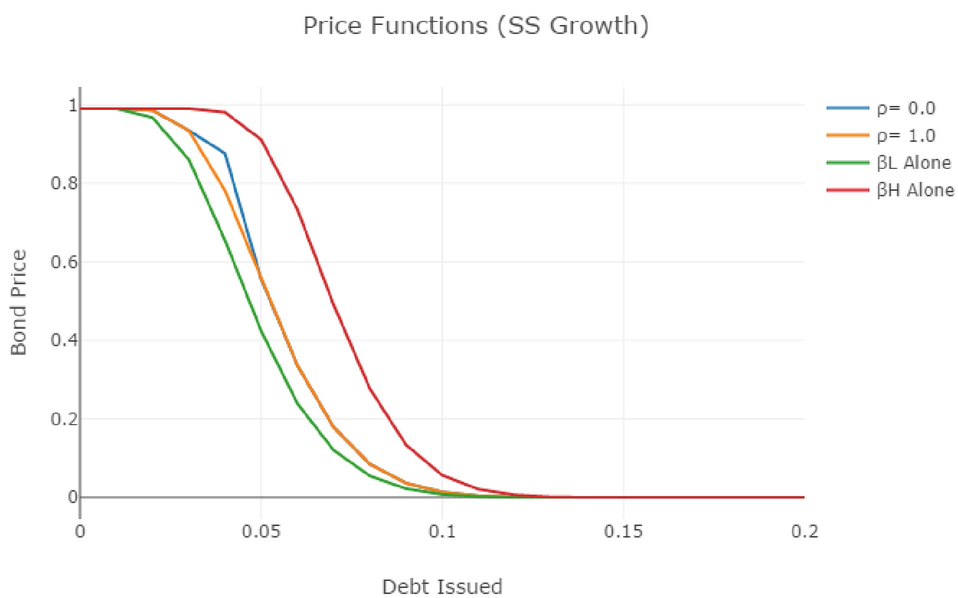

Figure 1:

Figure 1 also contains the pricing functions for the benchmark model, to which we will return. The equilibrium we compute turns out to be a hybrid-pooling equilibrium. The sovereign types pool their borrowing behavior but separate in default.

First, we look at the fiscal pooling. This is seen in the policy function in Figure 2, which shows that more borrowing occurs the higher is the growth rate, but the two types always borrow the same.

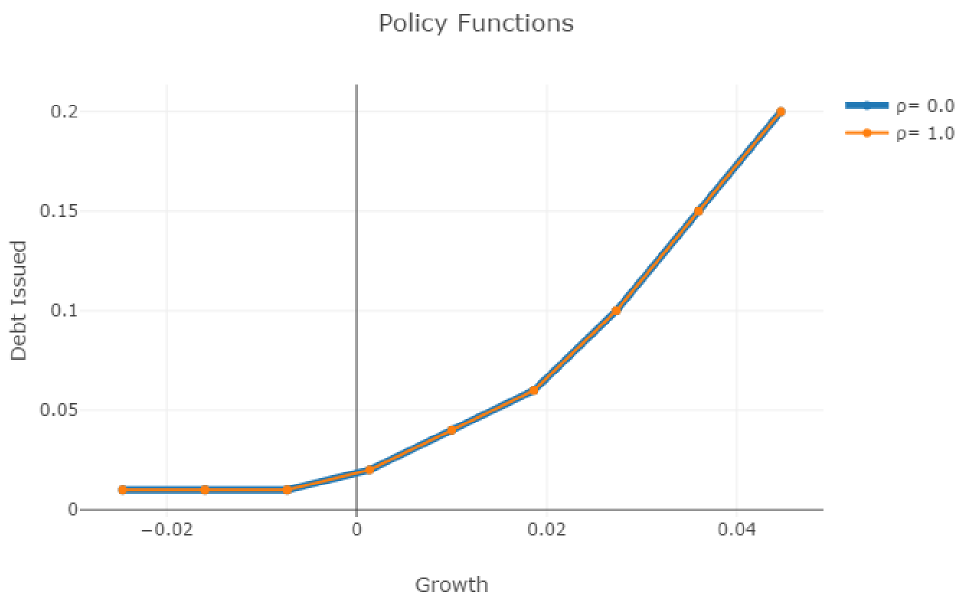

Figure 2:

Investors don't get to observe either type, so they cannot discriminate across types (they clearly would if they could). Given this, they must offer both types a similar pricing schedule, differentiated only by their beliefs. This schedule naturally falls in between the two types, as seen in Figure 1. This clearly helps the low-type, who enjoys better prices in all states and hurts the high-type, who receives lower prices in all states.

This is interesting for a couple of reasons:

(1) The bad type actually has to borrow *more* to pool behavior. It's obvious why he wants to do this. In fact, this is in a sense the opposite of a single-crossing property. This may be why such equilibria are relatively easy to compute

(2) In equilibrium *the two types are made more similar*. By helping the bad type, he receives a better payoff from repayment, which causes him to default less often; in contrast, the good type receives a worse payoff from repayment, which causes him to default more often. Thus, the general equilibrium pricing dynamics *attenuate the difference in their behavior.*

This is why the pricing functions look different across beliefs, but not as different as each type in isolation. Even if the bad type reveals himself, he still defaults a lot less often than the case where he is the only type around because the prices are better, which fosters more repayment. The opposite is true of the good type, which gives him very little incentive to distinguish himself by, for instance, overborrowing. This reinforces the fiscal pooling.

Now, we show the separation in default. Figure 1 also highlights that the sovereign appears to get better prices the more strong lender beliefs are that he is the good type. Since pricing is competitive, this implies that the good type continues to default less often, despite this attenuation in behavior across types. Figure 3 confirms this across all beliefs at the equilibrium level of issuance. The annualized spread goes from 5% for the good type to roughly 15% for the bad type.

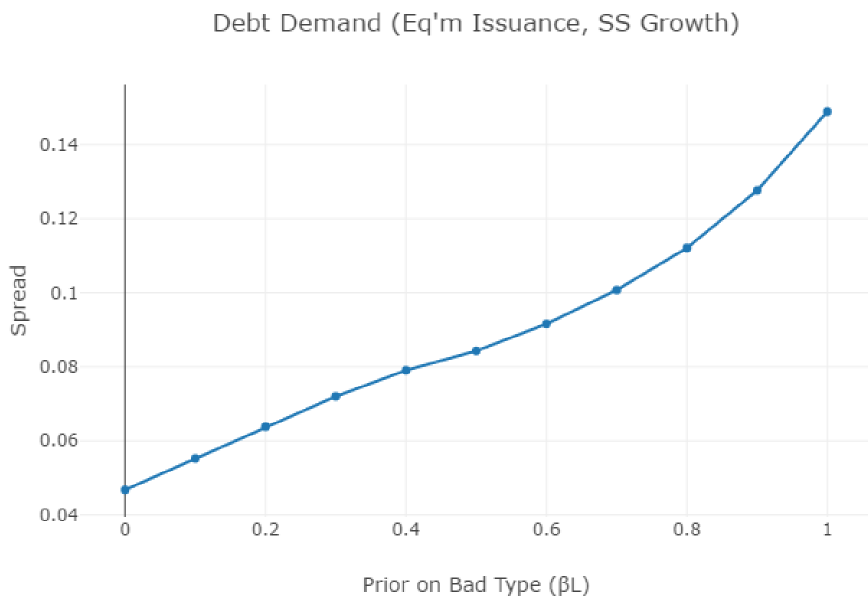

Figure 3:

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